MATH 453 : Numerical Methods III (4-0-4) 11/19/19

Catalog Description: Numerical solution of partial di erential equations. Numerical solution of boundary value problems and initial-value problems using nite di erence and other methods. Analysis of stability, accuracy, and implementations of methods.

Course Objectives: After completing this course, students will be able to

1. Derive and implement nite di erence stencils to numerically approximate solutions of elliptic, parabolic, and hyperbolic partial di erential equations in one and two dimensions.
2. Compute numerical solution approximations and compare with exact (known) solutions.
3. Compute convergence rates and computationally analyze the error of numerical approximation methods.

Learning Outcomes and Performance Criteria

1. Demonstrate an understanding of the terminology relevant to numerical solutions of partial di erential equations.

   Core Criteria:

   (a) De ne and explain the di erence between and ordinary and partial di erential equations.

   (b) Show how to classify a partial di erential equation as parabolic, elliptic, or hyperbolic and explain what these terms mean and how these classi cations a ect the solutions of such equations.

   (c) De ne at least one (physical) example of each of the three classes of partial di erential equations in one, two, and three dimensions.

   (d) Explain and give at least one example of the terms: Initial-value problem, Boundary value problem, Neumann and Dirichlet boundary conditions for both ordinary and partial di erential equations.

   (e) Characterize the propagation of numerically induced error. Identify \numerical di usion" and \numerical dispersion", for instance numerical smoothing of a non-smooth waveform.

   (f) Derive, explain, and demonstrate failure to account for the CFL (Courant-Friedrichs-Lewy) condition in both one and two dimensions.

2. Use Taylor series to create numerical routines (stencils) to approximate the solutions of partial di erential equations in one dimension.

   Core Criteria:

   (a) Derive and implement nite di erence stencil to approximate the solution to a one-dimensional, initial-value, boundary-value, heat equation over a given spacial domain for a given time interval with the following methods:

      Forward-time centered-space (explicit).

      Backward-time centered-space (implicit).

      The Crank-Nicolson method.

(b) Derive and implement the method of (explicit) centered nite di erences to create a series of snapshots of the solution of the following vibrating string problem: $u_{tt} = c^2 u_{xx}$ for $0 \leq x \leq a$, with $t = [0; T]$. The boundary and initial conditions are $u(x; 0) = f(x); \quad u_t(x; 0) = F(x); \quad u(0; t) = u(a; t) = 0$.

Additional Criteria:

(a) Derive and implement the method of nite volumes to create a series of snapshots of the solution of the following vibrating string problem: $u_{tt} = c^2 u_{xx}$ for $0 \leq x \leq a$, with $t = [0; T]$. The boundary and initial conditions are $u(x; 0) = f(x); \quad u_t(x; 0) = F(x); \quad u(0; t) = u(a; t) = 0$. Compare with solutions generated via nite di erences.

(b) Assorted applications using one-dimensional nite elements or spectral methods.

3. Use Taylor series to create numerical routines (stencils) to approximate the solutions of partial di erential equations in two dimensions.

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0, for all x and y on the (rectangular) boundary. Compare with solutions generated via nite di erence methods.

(c) Assorted applications using two-dimensional nite elements or spectral methods.

(d) Assorted examples with adaptive mesh re nement.

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