MATH 342 : Linear Algebra II (3-0-3) 10/17/18

**Catalog Description:** A continuation of the topics of MATH 341 to the setting of abstract vector spaces. Includes the study of orthogonality, inner product spaces, eigenvalues and eigenvectors, matrix decompositions and a more advanced study of linear transformations.

## Course Objectives:

- 1. Use orthogonality in  $\mathbb{R}^n$ .
- 2. Understand vector spaces.
- 3. Understand and work with the inner product.
- 4. Apply optimization algorithms to solve linear and non-linear problems.

# Learning Outcomes and Performance Criteria

- 1. Understand the signicance of orthogonality in  $\mathbb{R}^n$ .
  - Core Criteria:
  - (a) Find the representation of a vector with respect to an orthogonal or orthonormal basis by projection.
  - (b) Determine whether a set of vectors is orthogonal or orthonormal.
  - (c) Given a basis for a subspace of  $\mathbb{R}^n$ , nd a basis for the orthogonal complement of the space.
  - (d) Given a basis for a subspace W of  $\mathbb{R}^n$  and a  $\mathbf{v}$  in  $\mathbb{R}^n$ , nd the unique orthogonal decomposition  $\mathbf{v} = \mathbf{w} + \mathbf{w}^?$  with  $\mathbf{w}$  in W and  $\mathbf{w}^?$  in W?.
  - (e) Apply the Gram-Schmidt process to a set of vectors; nd the *QR* factorization of a matrix.
  - (f) Give the spectral decomposition of a symmetric matrix.
- 2. Understand and work with general vector spaces.

### Core Criteria:

- (a) Given a set of objects and de nitions of addition and scalar multiplication,
  - i. prove any of the properties of a vector space that hold,
  - ii. give speci c counterexamples to any properties that do not hold,
  - iii. if there is a zero vector, determine what it is; if all vectors have inverses, give the general form of the inverse of a vector,
  - iv. determine whether the set with those operations is a vector space.
- (b) Determine whether a subset of a vector space is a subspace.
- (c) Determine whether a set of vectors is linearly independent; if not, give one as a linear combination of the others.
- (d) Determine whether a set of vectors is a basis for a vector space; if not, tell why.
- (e) Give the representation of a vector with respect to a given basis.

- (f) Extend a set to a basis for a given space.
- (g) Reduce a spanning set to a basis.
- (h) Given the representation of a vector with respect to one basis, determine its representation with respect to another.
- (i) Determine the change-of-basis matrix from one basis to another.
- (j) Determine whether a given transformation is linear.
- (k) Given the action of a linear transformation on basis vectors, nd the linear transformation of any vector.
- (I) Determine whether a given vector is in the kernel or range of a linear transformation. Describe the kernel and range of a linear transformation.
- (m) Determine whether a given linear transformation is (a) one-to-one, (b) onto.
- (n) Determine whether two vector spaces are isomorphic. If they are, give an isomorphism from one to the other.
- (o) Determine the matrix of a linear transformation with respect to given bases.
- 3. Understand and work with the inner product.

#### Core Criteria:

- (a) Determine whether a given operation on a vector space is an inner product.
- (b) Compute the inner product of two vectors, norm of a vector, distance between two vectors. Determine whether two vectors are orthogonal.
- (c) Apply the Gram-Schmidt process to a set of orthogonal vectors to obtain an orthogonal basis.
- (d) Compute the least squares line or parabola for a set of data points. Compute the least squares solution to a system of equations. Solve problems involving least squares approximation.
- (e) Find the standard matrix of the approximation onto a subspace; nd the projection of a vector onto a subspace.
- (f) Find the singular values of a matrix; nd the singular value decomposition of a matrix.
- 4. Apply optimization algorithms to solve linear and non-linear problems.

### Core Criteria:

- (a) Find the maximum and/or minimum value of a quadratic form  $Q(\mathbf{x})$ , where  $\mathbf{x}$  ranges over a set of vectors that satisfy some constraint using eigenvalues and eigenvectors.
- (b) Use matrix decomposition (such as LU, QR, SVD, Cholesky decomposition) to solve optimization problems.
- (c) Use Gradient descent methods to solve optimization problems.
- (d) Be able to check conditions under which di erent optimization techniques are appropriate for a given problem.